

Global Monopole-BTZ black hole

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In order to obtain the geometry of a global monopole we introduce the broken $O(2)$ symmetry in $2+1$ -dimensions. Adding a negative cosmological constant yields the extension of BTZ black hole in a global monopole background. In the absence of an exact solution and to the first order of approximation the global monopole adds $\frac{1}{r^2}$ and $\ln r$ potentials to the metric functions.

Keywords: 2+1-dimensions; Global monopole; BTZ black hole;

I. INTRODUCTION

The analogue of Barriola-Vilenkin's global monopole spacetime [1] in $3+1$ -dimensions is constructed in $2+1$ -dimensions. Considerable attention received by the lower dimensions during recent decades provides the main motivation for such a study. It is not only that it constitutes a test bed for higher dimensions but $2+1$ -dimensions can also be considered as a brane in $3+1$ -dimensions. Historically the idea was popularized first by the $2+1$ -dimensional Banados-Teitelboim-Zanelli (BTZ) black hole solution [2, 3] which was sourced by a negative cosmological constant. In other words, the absence of gravitational degrees of freedom in lower dimensions was filled by a cosmological constant. Now, the similar role will be played by both a cosmological constant and a global monopole together. We refer to such a spacetime as the Global-Monopole-BTZ (GMBTZ) spacetime. The global monopole is localized in a core whose effect remains asymptotically much weaker relative to the cosmological constant term.

The global monopole in $3+1$ -dimensions has the symmetry group $O(3)$ to be broken spontaneously to $U(1)$. The similar role is played in the $2+1$ -dimensional case by the abelian group $O(2)$. Instead of a triplet of scalar fields we have now a doublet of scalar fields $\phi^a = \eta f(r) \frac{x^a}{r}$, (for $a = 1, 2$), with η = monopole charge constant, $f(r)$ a radial function to be determined and $(x^a)^2 = r^2$. The differential equation satisfied by $f(r)$ can't be solved exactly, however, far beyond the monopole's core we can set $f(r) \simeq 1$. We show that this is possible in flat space to the order $\sim \frac{1}{r^2}$ and the same trend follows also in the curved spacetime. This makes a black hole whose horizon depends strongly on the monopole charge and the cosmological constant. Interestingly, the geometry obtained for such a monopole with $f(r) \simeq 1$ is reminiscent of the geometry of a charged-BTZ (CBTZ) black hole [4–6]. In other words, the geometry doesn't differentiate asymptotically whether the agent source is an external field of a global monopole or the electric charge of a central black hole. That is, global monopoles which are

believed to emerge from spontaneous symmetry breaking during the big bang [7–9] plays pretty well the role of an electrostatic field.

More generally the choice $f(r) \simeq 1 + \frac{f_1}{r^2}$, with $f_1 = \text{const.}$, we have both a logarithmic term $\sim \ln r$ and a term $\sim \frac{1}{r^2}$ so that the analogy with the CBTZ black hole spacetime is no more valid.

II. GLOBAL MONOPOLE IN $2+1$ -DIMENSIONS

We start with the general form of static, circularly symmetric line element in $2+1$ -dimensions given by

$$ds^2 = -A(r) dt^2 + \frac{1}{B(r)} dr^2 + r^2 d\theta^2 \quad (1)$$

in which $A(r)$ and $B(r)$ are two functions only of r . Now, we introduce the action consisting of a doublet of real scalar fields of the form ($16\pi G = c = 1$)

$$S = \int d^3x \sqrt{-g} (R - 2\Lambda + L^{field}) \quad (2)$$

in which

$$L^{field} = -\frac{1}{2} \partial_\mu \phi^a \partial^\mu \phi^a - \frac{1}{4} \lambda (\phi^a \phi^a - \eta^2)^2. \quad (3)$$

Here $a = 1, 2$, R is the Ricci scalar, λ is a coupling constant, Λ is the cosmological constant, η is the symmetry-breaking scale parameter and

$$\phi^a = \eta f(r) \frac{x^a}{r}, \quad (4)$$

for $x^1 = r \cos \theta$ and $x^2 = r \sin \theta$. To find the field equation for $f(r)$ we express the field Lagrangian in terms of $f(r)$ only, i.e.,

$$L^{field} = -\frac{\eta^2 B}{2} f'^2 - \frac{\eta^2}{2r^2} f^2 - \frac{1}{4} \lambda \eta^4 (f^2 - 1)^2. \quad (5)$$

Now, variation of the action with respect to f yields

$$f'' + \left(\frac{1}{r} + \frac{1}{2AB} (AB)' \right) f' - \left(\frac{1}{r^2} + \lambda \eta^2 (f^2 - 1) \right) \frac{f}{B} = 0 \quad (6)$$

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in which a prime stands for the derivative with respect to r . Variation with respect to $g^{\mu\nu}$ yields the Einstein equations

$$G_\mu^\nu + \frac{1}{3}\Lambda\delta_\mu^\nu = T_\mu^\nu \quad (7)$$

in which

$$T_\mu^\nu = \frac{1}{2} \left(\partial_\mu \phi^a \partial^\nu \phi^a - \frac{1}{2} \partial_\rho \phi^a \partial^\rho \phi^a \delta_\mu^\nu \right) - \frac{1}{8} \lambda (\phi^a \phi^a - \eta^2)^2 \delta_\mu^\nu. \quad (8)$$

An explicit calculation gives

$$T_t^t = -\frac{\eta^2}{4} \left(B f'^2 + \frac{1}{r^2} f^2 + \frac{\lambda}{2} \eta^2 (f^2 - 1)^2 \right), \quad (9)$$

$$T_r^r = \frac{\eta^2}{4} \left(B f'^2 - \frac{1}{r^2} f^2 - \frac{\lambda}{2} \eta^2 (f^2 - 1)^2 \right) \quad (10)$$

and

$$T_\theta^\theta = -\frac{\eta^2}{4} \left(B f'^2 - \frac{1}{r^2} f^2 + \frac{\lambda}{2} \eta^2 (f^2 - 1)^2 \right). \quad (11)$$

In addition, the Einstein tensor's components are given by

$$G_t^t = \frac{1}{2} \frac{B'}{r}, \quad (12)$$

$$G_r^r = \frac{1}{2} \frac{A'B}{rA} \quad (13)$$

and

$$G_\theta^\theta = \frac{1}{4} \frac{2A''AB - A'^2B + A'B'A}{A^2}. \quad (14)$$

We note that, as in 3+1-dimensional case, the size of the global monopole in flat spacetime is given by $\delta = \frac{1}{\eta\sqrt{\lambda}}$. In the case of the flat spacetime $B = 1 = A$ and the field equation for f becomes

$$f'' + \frac{1}{r} f' - \left(\frac{1}{r^2} + \lambda \eta^2 (f^2 - 1) \right) f = 0. \quad (15)$$

An exact solution is not available for (15) but imposing the condition that $\lim_{r \rightarrow \infty} f = f_{finite} = c_0$ helps us to expand $f(r)$ for large r as

$$f = c_0 + \frac{c_1}{r} + \frac{c_2}{r^2} + \dots \quad (16)$$

in which $c_i = \text{const}$. A direct substitution in (15) implies $c_0 = 1$, $c_1 = 0$ and $c_2 = -\frac{1}{2\lambda\eta^2}$ and therefore up to r^{-2} one finds

$$f \simeq 1 - \frac{1}{2\lambda\eta^2 r^2}. \quad (17)$$

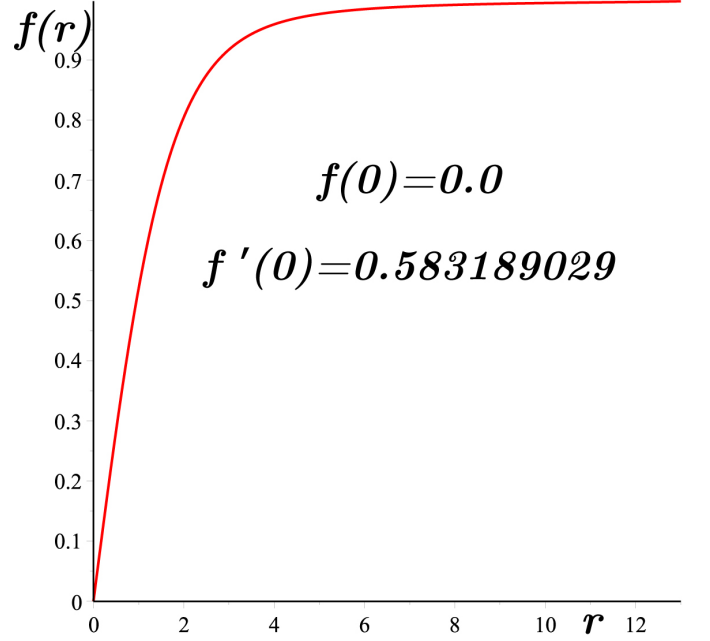


FIG. 1: A plot of the solution to the field equation (6) $f(r)$ with respect to r for $\lambda = 1 = \eta$ in flat spacetime. The initial conditions are set in such a way that $f(r)$ asymptotically approaches to one. This is in agreement with the series solution found for $f(r)$ in which outside the core where $r \gg \delta$ the solution up to the second order is given by $f \simeq 1 - \frac{1}{2\lambda\eta^2 r^2}$.

Let's add that a numerical solution for the field equation (15) with the boundary conditions $f(0) = 0$, $\lim_{r \rightarrow \infty} f(r) = 1$ and $\lambda = 1 = \eta$ is displayed in Fig. 1. Let us also note that from the numerical analysis setting $f'(0) = 0.583189029$ is crucial, otherwise the solution does not approach to 1 asymptotically. Our numerical solution has similar counterpart in 3 + 1-dimensional flat spacetime [10, 11]

Therefore for the flat spacetime one concludes that for large r which is outside the core of the monopole i.e. $r \gg \delta$, a good estimation for f is $f \simeq 1$. As we shall see in the next section, for the case of curved spacetime due to a global monopole we also abide by the same assumption namely, for large r still $f \simeq 1$. Consequently the energy momentum tensor for the flat spacetime is given by

$$T_\mu^\nu = \text{diag} \left[-\frac{\eta^2}{4r^2}, -, \frac{\eta^2}{4r^2}, \frac{\eta^2}{4r^2} \right]. \quad (18)$$

A. Global monopole-BTZ (GMTZ) solution

To find solutions to the Einstein equations, let's first combine the tt and rr components of the Einstein's equations. This yields

$$\frac{A'}{A} - \frac{B'}{B} = r\eta^2 f'^2 \quad (19)$$

whose integration implies

$$A = B \exp \left(\eta^2 \int r f'^2 dr + C \right). \quad (20)$$

in which C is an integration constant. Next, we consider the tt component equation which admits

$$-B' = \frac{2\Lambda r}{3} + \frac{\eta^2}{2r} + \frac{\eta^2 r}{2} \left(B f'^2 + \frac{1}{r^2} (f^2 - 1) + \frac{\lambda}{2} \eta^2 (f^2 - 1)^2 \right). \quad (21)$$

The exact solution for the latter equation is not possible. We assume that f behaves at infinity according to

$$f = c_0 + \frac{c_1}{r} + \frac{c_2}{r^2} + \frac{c_3}{r^3} + \frac{c_4}{r^4} + \mathcal{O} \left(\frac{1}{r^5} \right) \quad (22)$$

and we find the solution for $B(r)$. Next, we expand $B(r)$ about infinity up to 4th inverse power and after that we apply the same for $A(r)$ which is given in (20). Finally we substitute the expanded form of f , B and A into the other two equations. Up to 4th order we find

$$\begin{aligned} c_0 &= 1, \\ c_1 &= c_3 = 0 \end{aligned} \quad (23)$$

and

$$c_2 = -\frac{1}{2\lambda\eta^2}. \quad (24)$$

Consequently the explicit form of A and B up to second order are found as

$$A(r) \simeq B(r) = \frac{r^2}{\ell^2} - M - \frac{1}{2} \eta^2 \ln(r) + \frac{2 - \ell^2 \eta^2 \lambda}{8\eta^2 \ell^2 \lambda^2 r^2} + \mathcal{O} \left(\frac{1}{r^4} \right) \quad (25)$$

with

$$f \simeq 1 - \frac{1}{2\lambda\eta^2 r^2} + \mathcal{O} \left(\frac{1}{r^4} \right). \quad (26)$$

For zero cosmological constant $\ell^2 \rightarrow \infty$ and we obtain $A \sim B = -M - \frac{1}{2} \eta^2 \ln r - \frac{1}{8\lambda r^2} + \mathcal{O} \left(\frac{1}{r^4} \right)$. We note that, up to second order, the energy momentum tensor is as given in Eq. (18) but the next order brings new terms such that

$$T_\mu^\nu \simeq \text{diag} \left[-\frac{\eta^2}{4r^2} + \frac{\Delta_0}{r^4}, -\frac{\eta^2}{4r^2} + \frac{\Delta_1}{r^4}, \frac{\eta^2}{4r^2} + \frac{\Delta_2}{r^4} \right] \quad (27)$$

where $\Delta_0 = \frac{\ell^2 \eta^2 \lambda - 2}{8\eta^2 \ell^2 \lambda^2}$, $\Delta_1 = \frac{\ell^2 \eta^2 \lambda + 2}{8\eta^2 \ell^2 \lambda^2}$ and $\Delta_2 = \frac{3\ell^2 \eta^2 \lambda + 2}{8\eta^2 \ell^2 \lambda^2}$. If one neglects the higher powers in T_μ^ν it reduces to the case $f \simeq 1$ then the solution becomes

$$A(r) \simeq B(r) = \frac{r^2}{\ell^2} - M - \frac{1}{2} \eta^2 \ln(r) + \mathcal{O} \left(\frac{1}{r^2} \right). \quad (28)$$

This solution is isometric to the CBTZ [4–6] black hole solution whose electric charge is replaced by the global monopole parameter (charge) η . This solution, therefore, can appropriately be called as the Global-Monopole-BTZ (GMBTZ) solution which is valid to the order $\frac{1}{r^2}$.

III. CONCLUSION

We obtain the metric of a global, chargeless monopole in 2+1–dimensions which is the analogue of the Barriola-Vilenkin's monopole in 3+1–dimensions [1]. Outside the core of the global monopole we have an asymptotic black hole solution with the hair of a monopole. This may be attributed to the topological remnants of a 2+1–dimensional big-bang where $O(2)$ is the spontaneously broken symmetry although the results are much similar to its 3+1–dimensional counterparts. The global monopole modifies the BTZ black hole whose Hawking temperature becomes strongly dependent on the monopole parameter. When compared with the CBTZ black hole we observe that in 2+1–dimensions the global monopole charge with $f(r) \simeq 1$ plays the similar role of electric charge in the CBTZ black hole solution. We must add finally that the absence of an exact solution to the global monopole differential equation (Eq. (6)) in 2+1– and also in higher dimensions is a serious toward against a complete understanding of the global monopole problem in a curved spacetime.

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